



ST. JOSEPH'S COLLEGE, PRAYAGRAJ

HALF YEARLY EXAM – OCTOBER 2023

MATHEMATICS

Time: 2½ Hrs.

Class X

M.M.: 80

Answers to this Paper must be written on the paper provided separately.

You will not be allowed to write during the first 15 minutes.

This time is to be spent in reading the question paper.

The time given at the head of this Paper is the time allowed for writing the answers.

Attempt all questions from Section A and any four questions from Section B
All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of questions are given in brackets []

Mathematical tables are provided.

SECTION A

(Attempt all questions from this Section)

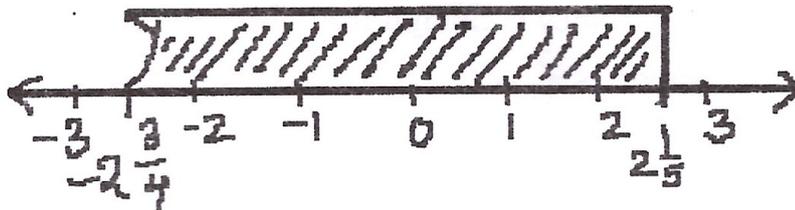
Question 1. Choose the correct answers to the questions from the given options. [15]

i. Mr. Das deposits Rs. 240 per month for 2 years in a cumulative deposit account.

If the rate of interest is 10% p.a. simple interest, then the interest per month is

- a. Rs. 6000.00 b. Rs. 600.00 c. Rs. 500.00
d. Rs. 250.00 e. None of these

ii. The replacement set of the below given number line is



- a. $\{x: -2\frac{3}{4} < x < 2\frac{1}{5}; x \in R\}$ b. $\{x: -2\frac{3}{4} \leq x \leq 2\frac{1}{5}; x \in R\}$
c. $\{x: -2\frac{3}{4} < x \leq 2\frac{1}{5}; x \in R\}$ d. $\{x: -2\frac{3}{4} \leq x < 2\frac{1}{5}; x \in R\}$

iii. The product AB of two matrices A and B is possible if

- a. A and B have same number of rows b. A and B have same number of columns
c. The number of columns of A is equal to the number of rows of B.
d. The number of rows of A is equal to the number of columns of B.

iv. The value(s) of m for which the equation $x^2 + mx + (m + 3) = 0$ has equal roots is(are):

- 1) -6 2) 2 3) -6 4) -2
a. Only 1 b. Both 1 and 4 c. Only 2 d. Both 2 and 3.

v. A trader sells an article to a consumer for Rs. 5600.00 (inclusive of tax). If the rate of GST is 12%, then the taxable amount is

- a. Rs. 5000.00 b. Rs. 6272.00 c. Rs. 5936.00 d. Rs. 4832.00





- vi. If the degree of the divisor is 2, then the possible degree of the remainder will be
a. Only 0 b. Only 1 c. Only 2 d. Either 0 or 1
- vii. If $k - 1, k + 1$ and $2k + 3$ are in A.P. then the value of k is
a. -2 b. 0 c. 2 d. 4 e. None of these
- viii. If Tarun invests Rs. 19200 on Rs. 50 shares at a premium of 20%, then the number of shares he buys is
a. 640 b. 384 c. 320 d. 160 e. None of these
- ix. The centroid of the triangle whose vertices are $(3, -7), (-8, 6)$ and $(5, 10)$ is
a. $(0, 9)$ b. $(0, 3)$ c. $(1, 3)$ d. $(3, 3)$ e. None of these
- x. A : Two players Yuvraj and Mazin play a chess match. It is known that the probability of Yuvraj winning the match is 0.47, then the probability of Mazin losing the game will be 0.53.
R : $P(\text{Yuvraj losing}) + P(\text{Mazin losing}) = 1$
a. A is true but R is false
b. A is false but R is true
c. Both A and R are true
d. Both A and R are false
- xi. A point P is reflected about the x-axis followed by the reflection about the y-axis. If the coordinate of the reflected point is $(-5, 7)$. The coordinate of the point P is
a. $(5, 7)$ b. $(-5, 7)$ c. $(-5, -7)$ d. $(5, -7)$
- xii. The fourth proportional to 3, 4, 5 is
a. 6 b. $\frac{20}{3}$ c. $\frac{15}{4}$ d. $\frac{12}{5}$ e. None of these
- xiii. $(\sin A + \cos A)^2 + (\sin A - \cos A)^2$ is
a. 4 b. 3 c. 2 d. 1 e. None of these
- xiv. A man observes the top of a chimney 60m high from the ground, such that the tangent of the angle of elevation is $\frac{12}{5}$. The distance of the man from the foot of the chimney is
a. 10m b. 15m c. 20m d. 25m e. None of these
- xv. The slope of a line parallel to y - axis is
a. 0 b. 1 c. -1 d. Not defined

Question 2.

- i. Saubhagya deposits Rs. 700 in a recurring deposit account in a nationalised bank for $3\frac{1}{4}$ years. If the rate of interest is 5.5% p.a. simple interest. Calculate:
a. The interest received
b. The maturity value [4]
- ii. If $2x^3 + ax^2 + bx - 14$ has a factor $(2x + 7)$ and leaves a remainder 52 when divided by $(x - 3)$. Find the values of a and b . [4]
- iii. A straight line passes through the points $(-1, 4)$ and $(5, -2)$. It intersects the coordinate axes at points A and B. M is the point on AB that divides AB in the ratio 1 : 2. Find the equation of a line passing through M and perpendicular to AB. [4]

Question 3.

i. Prove that: $\frac{\cos A}{(1-\tan A)} + \frac{\sin A}{(1-\cot A)} = (\sin A + \cos A)$ [4]

ii. If a, b, c & d are in continued proportion, prove that:

$$\sqrt{ab} + \sqrt{bc} - \sqrt{cd} = \sqrt{(a+b-c)(b+c-d)}$$
 [4]

iii. Use graph for the question. Take $2\text{cm} = 1$ unit on both the axes. Plot points $A(2, -3), B(5, 2), C(1, 2), D(2, 4)$ and $E(0, 8)$

- Reflect the points A, B, C and D about the y -axis and label them as A', B', C' and D' respectively. Also write their coordinates.
- Name the invariant point(s) when reflected about the y -axis.
- Give the geometrical name of the closed figure $ABCDE D' C' B' A'$.
- Write the equation of the line of symmetry. [5]

SECTION B

(Answer any four questions from this section)

Question 4.

i. If $\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} M = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$, find the matrix M . [3]

ii. Find the total amount to be paid by the customer for the following bill. [3]

Item	MRP	Discount	GST
Ear buds	Rs. 3000	10%	12%
Fitness Band	Rs. 4000	15%	18%

iii. ABCD is a rhombus $O(1, 4)$ is the point of intersection of two diagonals. Coordinates of A are $(-1, 2)$, find [4]

- Coordinates of C
- Slope of diagonal AC
- Equation of the other diagonal BD .

Question 5.

i. Which term of the sequence $6, 2, \frac{2}{3}, \dots$ is $\frac{2}{243}$ [3]

ii. If $P(9a - 2, -b)$ divides the line segment joining $A(3a + 1, -3)$ and $B(8a, 5)$ in the ratio $3 : 1$, find the values of a and b . [3]

iii. A vertical pole and vertical tower are on the same level ground. From the top of the pole the angle of elevation of the top of the tower is 60° and the angle of depression of the foot of the tower is 30° . Find the height of the tower if the height of the pole is 20m . [4]

Question 6.

i. Vipul invests Rs. 3750.00 in a company paying 10% dividend when its Rs. 10.00 shares can be bought for Rs. 12.50. Find [3]

- The number of shares bought by him
- His annual income from these shares
- The return percentage.

ii. Solve the equation $3x^2 - x - 7 = 0$ and give your answer correct to two significant figures. [3]

- iii. Using factor theorem, factorise the given expression completely: [4]
 $x^3 - 3x^2 - 10x + 24.$

Question 7.

- i. Solve the following linear inequation and represent the solution set on the number line. [3]
 $\frac{1}{2}(2x - 1) \leq 2x + \frac{1}{2} \leq 5\frac{1}{2} + x; x \in R$
- ii. Prove that: $\frac{\tan^2 A}{\tan^2 A + 1} + \frac{\operatorname{cosec}^2 A - 1}{1 + \cot^2 A} = 1$ [3]
- iii. The sum of the numerator and denominator of a fraction is 8. If 1 is added to both the numerator and denominator, the fraction is increased by $\frac{1}{15}$. Find the fraction. [4]

Question 8.

- i. Find two numbers such that the mean proportion between them is 36 and the third proportion to them is 288. [3]
- ii. The 4th term of an A.P. is 11. The sum of the fifth and seventh terms of the A.P. is 34. Find its common difference. [3]
- iii. A box contains cards bearing numbers 6 to 70. If one card is drawn at random from the box, find the probability that it bears. [4]
- a. A one – digit number
 - b. A number divisible by 5
 - c. An odd number less than 30.
 - d. A composite number between 50 and 70.

*****ALL THE BEST*****